

Thermodynamic of the Ghost Dark Energy Universe

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Recently, the vacuum energy of the QCD ghost in a time-dependent background is proposed as a kind of dark energy candidate to explain the acceleration of the Universe. In this model, the energy density of the dark energy is proportional to the Hubble parameter H , which is the Hawking temperature on the Hubble horizon of the Friedmann-Robertson-Walker (FRW) Universe. In this paper, we generalized this model and choice the Hawking temperature on the so-called trapping horizon, which will coincides with the Hubble temperature in the context of flat FRW Universe dominated by the dark energy component. We study the thermodynamics of Universe with this kind of dark energy and find that the entropy-area relation is modified, namely, there is an another new term besides the area term.

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I. INTRODUCTION

Current observations converge on the fact that the Universe is accelerated expanding, lots of theoretical models were proposed to explain this phenomenology. In some of them, people proposed a new kind of dark energy component with negative pressure in our Universe, which will drive the acceleration, and the simplest dark energy model is the cosmological constant, but it suffers fine-tuning and coincidence problems. While, in other models, people try to modify the Einstein gravity at large scale in the Universe, e.g. $f(R)$, DGP, etc. models, then the Universe can be accelerated without introducing dark energy.

Recently, a very interesting dark energy model called Veneziano ghost dark energy has been proposed [1–4], and in this model, one can obtain a cosmological constant of just the right magnitude to give the observed expansion from the contribution of the ghost fields, which are supposed to be present in the low-energy effective theory of QCD without introducing any new degrees of freedom. The ghosts are needed to solve the $U(1)$ problem, but they are completely decoupled from the physical sector [5]. The ghosts make no contribution in the flat Minkowski space, but make a small energy density contribution to the vacuum energy due to the off-set of the cancellation of their contribution in curved space or time-dependent background. For, example, in the Rindler space, the contribution of high frequency modes is suppressed by the factor $e^{-2\pi k/a_T}$ and the main contribution comes from $k \sim a_T$, where a_T is the temperature on the horizon seen by the Rindler observer [6, 7]. In the cosmological context, one can choice $a_T \sim H$, which corresponds to the temperature on the Hubble horizon. Then, in the context of strongly interacting confining QCD with topological nontrivial sector, this effect occurs only in the time direction and their wave function in other space directions is expected to have the size of QCD energy scale. Thus, this ghost gives the vacuum energy density proportional to $\Lambda_{QCD}^3 H_0$. With $H_0 \sim 10^{-33}\text{eV}$, it gives the right order of observed magnitude $\sim (3 \times 10^{-3}\text{eV})^4$ of the energy density.

It should be emphasized that the Veneziano ghost from the ghost dark energy model is not a new propagating physical degree of freedom and the description of dark energy in terms of the Veneziano ghost is just a matter of convenience to describe very complicated infrared dynamics of strongly coupled QCD and it does not violate unitarity, causality, gauge invariance and other important features of renormalizable quantum field theory, see [8–11]. Generally, it is very difficult to accept the linear behavior that the energy of FRW Universe is linear in Hubble constant " H ", because QCD is a theory with a mass gap determined by the energy scale 100MeV. So, it is generally expected that there should be an exponentially small corrections rather than that linear corrections H . However, as the arguments discussed in refs.[8–11] that the linear scaling H is due to the complicated topological structure of strongly coupled QCD, not related to the physical massive propagating degree of freedom. Therefore, the linear in Hubble constant H scaling has a strong theoretical support tested in a number of models. The recent progress on the GDE model, see Ref. [12–14], in which the author also consider other possibilities of the energy density form of the GDE model with

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or without interactions and also its thermodynamical behaviors.

However, there are some other horizons in the context of the FRW Universe, such as the future inner trapping horizon, which will be defined in the next section. One can also use the outer trapping horizon to define black holes in a general spacetime including time-dependent spacetime. The Hawking radiation of apparent horizon in a FRW Universe was firstly studied in Ref.[15]. In this paper, we will choose $a_T \sim T_t$, where T_t is the temperature defined on the trapping horizon of the FRW Universe. We study its thermodynamic behavior, and find that there is a new term contributes to the entropy-area relation.

On the other side, there exists some kind of correspondence between thermodynamical laws and gravitational equations. We will give a briefly review on this topic in the following section. In this paper, we assume that the thermodynamic laws as the origin of all the dark energy and modification gravity models, then we do the comparison between the ghost dark energy model (including the generalized ones) and DGP model, and find they are very similar but still have differences from the thermodynamical point of view.

This paper is organized as follows: In Sec. II, we give a briefly review on the unified first law of thermodynamics on the “inner” trapping horizon. In Sec. III, we study the thermodynamics of the ghost dark energy Universe and some generalized cases on the trapping horizon. Also we get the property of the generalized second law of thermodynamics of this model in this section. In the last section, we will give some discussions and conclusions.

II. BRIEFLY REVIEW ON THE UNIFIED FIRST LAW

In this section, we will give a brief review on the “unified first law” in the 3 + 1-dimensional spherical symmetric spacetime. This “unified law” was first proposed by Hayward [16–19], and it is a thermodynamical description of Einstein equations. It indicates that there could be some relations between thermodynamic laws and gravitational equations. And it seems that the gravitational theory maybe not a fundamental theory. Then, one can just use the thermodynamic laws to describe the gravity. For convenient, we will use the double-null form of the metric, namely, $ds^2 = -2e^{-f}d\xi^+d\xi^- + r^2d\Omega^2$. Here, $d\Omega^2$ is the line element of the 2-sphere with unit radius, r and f are functions of (ξ^+, ξ^-) . Each symmetric sphere has two preferred normal directions, namely the null directions $\partial/\partial\xi^\pm$, which will be assumed future-pointing in the following. And also, we will assume the spacetime is time-orientable.

The expansions of the radial null geodesic congruence are defined by $\theta_\pm = 2r^{-1}\partial_\pm r$, and ∂_\pm denotes the coordinates derivative along ξ^\pm . The expansion measures whether the light rays normal to the sphere are diverging ($\theta_\pm > 0$) or converging ($\theta_\pm < 0$), namely, whether the sphere is increasing or decreasing in the null directions. It should be noticed that the sign of θ_\pm will not change with geometries, while its value will. The only invariants of the metric and its first derivative are functions of r and $e^f\theta_+\theta_-$, or equivalently $g^{ab}\partial_a r\partial_b r = -\frac{1}{2}e^f\theta_+\theta_-$, which has an important physical and geometrical meaning: a sphere is said to be trapped (untrapped), if $\theta_+\theta_- > 0$ ($\theta_+\theta_- < 0$). And it is called a marginal sphere if $\theta_+\theta_- = 0$.

In the case of non-stationary black holes, Hayward [16] has proposed that the future outer trapping horizons defined as the closure of a hypersurface foliated by future or past, outer or inner marginal sphere is taken as the definition of black holes, since the horizon possess various properties which are often intuitively ascribed to black hole including confinement of observers and analogues of the zeroth, first and second law of thermodynamics. However, in the case of FRW Universe, one should take the future inner trapping horizon defined by [20–27], $\theta_+ = 0$, $\theta_- < 0$, $\partial_-\theta_+ > 0$ as a system on which the thermodynamics will be established, since the surface gravity is negative on the cosmological horizon.

The $(0, 0)$ component of Einstein equations could be written as a “unified first law” on this future inner trapping horizon as [20]

$$\langle dE, z \rangle = \frac{\kappa}{8\pi G} \langle dA, z \rangle + \langle W dV, z \rangle, \quad (1)$$

where E is defined as the Minsner-Sharp energy, $A = 4\pi r^2$ and $V = \frac{4\pi}{3}r^3$. Here, we have defined two invariants constructed from the energy-momentum tensor $T^{\mu\nu}$:

$$W = -\frac{1}{2}g_{ab}T^{ab} = -g_{+-}T^{+-}, \quad (2)$$

$$\Psi_a = T_a^b \partial_b r + W \partial_a r, \quad (3)$$

which are called the work density, and the energy flux vector (also called the energy-supply vector). Here and in the following, a, b denotes the two dimension space normal to the sphere. Note that, the Minsner-Sharp energy is a pure geometric quantity and has much better properties than the other definitions of energy when one consider the case of non-stationary spacetime. The relation between the Minsner-Sharp energy and others could be found in Ref. [16].

In Eq. (1), κ is defined by

$$\kappa = \frac{1}{2} \nabla^a \nabla_a r, \quad (4)$$

which is called the surface gravity of the trapping horizon, and z is a vector tangent to the trapping horizon [26, 27], which satisfies $z^a \partial_a (\partial_+ r) = 0$ on the trapping horizon. By taking the Einstein equations $\partial_+ \partial_+ r = -4\pi r T_{++}$, see Ref. [16] and the definition of the surface gravity in Eq. (4), one can easily finds

$$\langle A\Psi, z \rangle = \frac{\kappa}{8\pi G} \langle dA, z \rangle, \quad (5)$$

which is the Clausius relation in the version of black hole thermodynamics, see the first term on the right side of Eq. (1). The left side of the above equation is nothing but the heat flow δQ , and the right side has the form TdS , if one identifies the temperature $T = |\kappa|/2\pi$ and the entropy $S = A/4G$. So, in Einstein theory, the “unified first law” also implies the Clausius relation [20–27].

III. THERMODYNAMICS OF THE GHOST DARK ENERGY MODEL

In FRW Universe, one can also write the metric in the double-null form

$$ds^2 = -2d\xi^+ d\xi^- + \tilde{r}^2 d\Omega^2, \quad (6)$$

where $\tilde{r} = a(t)r$, and $a(t)$ is the scale factor. The definition of ξ^\pm are the same as that in [20–27]. By solving the equation $\partial_+ \tilde{r}|_{\tilde{r}=\tilde{r}_T} = 0$, we obtain the trapping horizon \tilde{r}_T as

$$\tilde{r}_T = \left(H^2 + \frac{k}{a^2} \right)^{-1/2}, \quad (7)$$

which coincides with the apparent horizon. The surface gravity is now $\kappa = -(1 - \epsilon)/\tilde{r}_T$, where we have defined $\epsilon \equiv \frac{\dot{\tilde{r}}_T}{2H\tilde{r}_T}$. It is easy to check that $\partial_- \tilde{r}_T < 0$, and the trapping horizon is future. Then, the project vector is given by $z = \partial_t - (1 - 2\epsilon)Hr\partial_r$, when $z^+ = 1$ is chosen in the (t, r) coordinates.

Actually, when one applies the first law to the apparent horizon to calculate the surface gravity and thereby the temperature by considering an infinitesimal amount of energy crossing the apparent horizon, the apparent horizon radius \tilde{r}_T should be regarded to have a fixed value [20], so in this sense, one has $\kappa \approx -1/\tilde{r}_T$. And also, one can see in the following, ϵ could be neglected when dark energy dominates the Universe. Therefore, the energy density of the ghost dark energy is given by as follows

$$\rho_{DE} = \alpha \sqrt{H^2 + \frac{k}{a^2}}, \quad (8)$$

where $\alpha > 0$ and is roughly of order of Λ_{QCD}^3 as we mentioned in the introduction section. Here, all the proportional coefficients are also absorbed in α , whose value could be given by observations, see the work of Cai et al. in Ref. [7].

The Friedmann equation reads

$$H^2 + \frac{k}{a^2} = \frac{1}{3} (\rho_{DE} + \rho_m), \quad (9)$$

where we have set $8\pi G = 1$. By solving the above equation and the continuity equations, we obtain the energy density and pressure of the dark energy

$$\rho_{DE} = \frac{\alpha^2}{6} \left(1 + \sqrt{1 + \frac{12\rho_m}{\alpha^2}} \right), \quad (10)$$

$$p_{DE} = \rho_m \left(1 + \frac{12\rho_m}{\alpha^2} \right)^{-1/2} - \rho_{DE}, \quad (11)$$

where we have neglected the $p_m \approx 0$ for matter. The equation of state parameter of the dark energy is

$$w_{DE} = \frac{p_{DE}}{\rho_{DE}} = -\frac{1}{2} \left[1 + \left(1 + \frac{12\rho_m}{\alpha^2} \right)^{-1/2} \right]. \quad (12)$$

When the matter decays $\rho_m \sim a^{-3} = (1+z)^3$, the dark energy will dominate the Universe, and its equation of state will trend to $w_{DE} \rightarrow -1$, see Eq. (12). And the present value of the equation of state is

$$w_{DE0} = -1 + \frac{\Omega_{m0}}{\Omega_{DE0} + 2\Omega_{m0}} + \frac{3\Omega_{m0}\Omega_{DE0}(\Omega_{m0} + \Omega_{DE0})}{(\Omega_{DE0} + 2\Omega_{m0})^3} z + o(z). \quad (13)$$

From the Friedmann equation (9), we also have

$$\alpha = 3H_0\Omega_{DE0}(\Omega_{DE0} + \Omega_{m0})^{-1/2}M_{pl}^2, \quad (14)$$

where $\Omega_{m0} = \rho_{m0}/(3H_0^2)$, $\Omega_{DE0} = \rho_{DE0}/(3H_0^2)$. Taking the values of $\Omega_{m0} \approx 0.27$ and $\Omega_{DE0} \approx 0.73$, one can get $w_{DE0} \approx -0.813 + 0.286z$ and $\alpha \approx (100 \text{ MeV})^3$. This values are consistent with the recent observations: $w_{DE0}^{obs} = -0.93_{-0.13}^{+0.13} + (0.41_{-0.72}^{+0.71})z$, (68%CL) in curved Universe ($k \neq 0$) from WMAP+BAO+H₀+SN [28].

By using the definition of ϵ and the Friedmann equation without approximation of $\kappa \sim 1/\tilde{r}_T$, we obtain the following relation

$$\epsilon = \frac{3}{4} \left(1 + \frac{w_{DE}}{1+r_c} \right), \quad r_c = \frac{\rho_m}{\rho_{DE}}. \quad (15)$$

For the detailed calculations, see Appendix A. From Eq. (15), it shows that when the dark energy dominates the Universe ($r_c \rightarrow 0$, $w_{DE} \rightarrow -1$), ϵ can be neglected. Then, in this sense our approximation for the energy density of the ghost dark energy (8) is also reasonable.

From Eqs. (2) and (3), we obtain the work density W_e and Ψ_e for the dark energy as

$$W_e = \rho_{DE} - \frac{\rho_m}{2} \left(1 + \frac{12\rho_m}{\alpha^2} \right)^{-1/2}, \quad (16)$$

$$\Psi_e = \frac{\rho_m}{2} \left(1 + \frac{12\rho_m}{\alpha^2} \right)^{-1/2} \left(-H\tilde{r}_T dt + a dr \right), \quad (17)$$

and then we have

$$\delta Q_{DE} = \langle A\Psi_{DE}, z \rangle = \frac{2\kappa A H \alpha^2 \epsilon}{3\rho_{DE}} = \frac{\pi \alpha^2}{3\rho_{DE}} T \langle dA, z \rangle, \quad (18)$$

where we have used the relation (A2) and (A3). Here, A denotes the surface area of the trapping horizon, namely, $A = 4\pi\tilde{r}_T^2$. For the heat flow of the pure matter ρ_m , we also have

$$\delta Q_m = \frac{\kappa}{8\pi G} \langle dA, z \rangle - \langle A\Psi_{DE}, z \rangle = 2\pi \left(1 - \frac{\alpha}{12\sqrt{\pi}} A^{1/2} \right) T \langle dA, z \rangle = T \langle dS_m, z \rangle, \quad (19)$$

where we have used the relation $A = 4\pi\alpha^2/\rho_{DE}^2$ from the Friedmann equation. Therefore, the entropy is given by

$$S_m = 2\pi A - \frac{\sqrt{\pi}\alpha}{9} A^{3/2}, \quad (20)$$

up to some integration constant. Here, the first term on the right hand side of the above equation is nothing but $A/4G$ when one recovers the induced Planck mass, while the second term is the additional term that becomes important when $A \gtrsim M_{pl}^4 \alpha^{-2} \sim H_0^{-2}$.

As we known, in the Dvali-Gabadadze-Porrati (DGP) model, the entropy is given by [29, 30]

$$S_m = \frac{A}{4G} \pm \frac{1}{24\sqrt{\pi}r_c G} A^{\frac{3}{2}}, \quad (21)$$

where $r_c = G_5/(2G)$ is the cross-over scale in the DGP model and G_5 is the 5-dimension gravitational constant in the bulk. Here, the minus sign in Eq. (21) corresponds to the self-accelerating branch of the DGP model, while plus sign corresponds to normal (non-accelerating) branch. Therefore, the entropies of the ghost dark energy model and DGP model (self-accelerating branch) are of the same order if $\alpha \sim 1/G_5$.

IV. GENERALIZED CASE

It is also interesting to consider higher order terms in the energy density of the dark energy, namely, Eq. (8) can be generalized to

$$\rho_{DE} = \alpha \sqrt{H^2 + k/a^2} + \beta(H^2 + k/a^2), \quad (22)$$

and when $\beta \rightarrow 0$, we recovered the model discussed in Sec. III. Eq. (22) was first proposed in Ref. [31–33] to get an accelerating Universe. By solving the Friedmann equation, we obtain

$$\rho_{DE} = \frac{3\alpha^2}{2(3-\beta)^2} \left(1 \pm \sqrt{1 + \frac{4(3-\beta)\rho_m}{\alpha^2}} \right) + \frac{\beta}{3-\beta}\rho_m, \quad (23)$$

for $\beta \neq 3$, while $\rho_{DE} = 3\alpha^{-2}\rho_m^2 - \rho_m$ for $\beta = 3$ (and it requires $\alpha < 0$). Then, in the case of $\beta \neq 3$, the associated work density W_e and Ψ_e for the dark energy are

$$W_e = \rho_{DE} - \frac{\rho_m}{2(3-\beta)} \left[\beta \pm 3 \left(1 + \frac{4(3-\beta)\rho_m}{\alpha^2} \right)^{-1/2} \right], \quad (24)$$

$$\Psi_e = \frac{\rho_m}{2(3-\beta)} \left[\beta \pm 3 \left(1 + \frac{4(3-\beta)\rho_m}{\alpha^2} \right)^{-1/2} \right] \left(-H\tilde{r}_T dt + adr \right), \quad (25)$$

and then we get

$$\delta Q_{DE} = \frac{\pi}{3} \left(\frac{\alpha A^{1/2}}{2\sqrt{\pi}} + 2\beta \right) T \langle dA, z \rangle, \quad (26)$$

and

$$\delta Q_m = 2\pi \left(1 - \frac{\beta}{3} - \frac{\alpha}{12\sqrt{\pi}} A^{1/2} \right) T \langle dA, z \rangle, \quad (27)$$

for the heat flow of the pure matter ρ_m . Therefore, the entropy is given by

$$S_m = 2\pi \left(1 - \frac{\beta}{3} \right) A - \frac{\sqrt{\pi}\alpha}{9} A^{3/2}, \quad (28)$$

up to some integration constant. From Eq. (28), one can see that the effect of β is to modify the slope of the entropy area relation in the first term of the right hand side of the above equation, while it does not contribute to the second term. In the case of $\beta = 3$, we have

$$W_e = -\frac{\rho_m}{2}, \quad \Psi_e = \left(3\alpha^{-2}\rho_m^2 - \frac{\rho_m}{2} \right) \left(-H\tilde{r}_T dt + adr \right), \quad (29)$$

and then

$$\delta Q_{DE} = 2\pi \left(1 - \frac{\alpha^2}{6\rho_m} \right) T \langle dA, z \rangle, \quad \delta Q_m = -\frac{\alpha\sqrt{\pi}}{6} A^{1/2} T \langle dA, z \rangle. \quad (30)$$

Therefore, the entropy is given by

$$S_m = -\frac{\sqrt{\pi}\alpha}{9} A^{3/2}, \quad (31)$$

up to some integration constant. Here, $\alpha < 0$, so the entropy increase with area.

We can also generalize the ghost dark energy model in a more general case with the energy density

$$\rho_{DE} = \sum_{n=-l}^m \alpha_n \left(H^2 + \frac{k}{a^2} \right)^{n/2}, \quad m, l \geq 0. \quad (32)$$

Although we can not obtain the exact relation between ρ_{DE} and ρ_m , we can still get the entropy area relation by using the same approach

$$S_m = 2\pi \left(1 - \frac{\alpha_2}{3}\right) A - \frac{16\pi^2}{3} \alpha_4 \ln A - \frac{2\pi}{3} \sum_{\substack{n=-l, \\ n \neq 0, 2, 4}}^m \frac{n\alpha_n}{4-n} (4\pi)^{\frac{n-2}{2}} A^{\frac{4-n}{2}}, \quad (33)$$

up to some integration constant. The logarithm term of the above equation could be also obtained from loop quantum cosmology [20], and usually, this term is regarded as a quantum correction. It should be noticed that the constant term α_0 (cosmological constant) do not contribute to the above entropy area relation.

Before ending this section, we would like to say something about the second law of thermodynamics of this model. The time derivative of entropy is given by

$$\dot{S}_m = 8\pi^2 H \tilde{r}_T^4 (\rho_m + p_m). \quad (34)$$

In our cases, we have neglected the pressure of matter $p_m \approx 0$, so we have $\dot{S}_m \geq 0$ which guarantees the second law of thermodynamics for our models. Furthermore, when dark energy dominates the Universe ($\rho_m \approx 0$), the entropy could be almost a constant depending on the parameters α_n .

V. DISCUSSION AND CONCLUSION

In this paper, we have studied the thermodynamics of the Universe with the Veneziano ghost dark energy component to drive the acceleration. By using the unified first law, we obtain the temperature and the corresponding entropy on the trapping horizon, which coincides with the apparent horizon in the FRW Universe. We find that there is a new term contributes to the entropy-area relation, see Eq. (20). This relation is very similar to that from the DGP model in the self-accelerating branch, so it can not distinguish these two models at this background evolution level, but it may be to distinguish them by doing the perturbation theory, and we will do the further studies on this subject.

We also generalized this model by adding a higher order term in Eq. (22). We solve the Friedmann equation exactly, and find the corresponding entropy area relation on the trapping horizon. Now, there are two new terms contribute to the relation. For a more generalized case, we have obtained the exact entropy area relation, which includes a logarithm term that could be also obtained from loop quantum cosmology and many terms with powers of area. However, there is no quadratic term $\sim A^2$, and the cosmological constant do not contribute this relation also.

As we have mentioned, there is a relation between gravity theory and thermodynamics, and gravity may be not a fundamental theory. Therefore, we can regard the thermodynamical laws as the first principle to get a physical model. For example, if we start from a general form of entropy area relation like Eq. (33), we can get the dark energy model with the same background equations as the generalized Veneziano ghost dark energy model, since there are could be some other models that have the same entropy area relation. But, one can still considers the thermodynamics laws as the origin of such kind of models.

Also, we find that the time derivative of entropy can not be negative, which means the second law of thermodynamics is hold in our models. Furthermore, If one assumes that some other components with equation of state $w < -1$ also exit in our models, then a generalized second law of thermodynamics is needed, see Ref.[29], and it deserves further studies.

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Appendix A: Calculation ϵ without approximation

Without the approximation $\kappa \sim 1/\tilde{r}_T$, the energy density of the dark energy is given by

$$\rho_{DE} = \frac{\alpha}{\tilde{r}_T} (1 - \epsilon), \quad (A1)$$

and from the Friedmann equation, we get

$$\dot{H} - \frac{k}{a^2} = -\frac{1}{2} \left[\rho_{DE}(1 + w_{DE}) + \rho_m \right], \quad (\text{A2})$$

while we also have the relation

$$\dot{H} - \frac{k}{a^2} = -\frac{2\epsilon}{\bar{r}_T^2}, \quad (\text{A3})$$

by using the definition of ϵ . From Eqs. (A2) and (A3), we get the following relation

$$\frac{4\epsilon}{3} = 1 + \frac{w_{DE}}{1 + r_c}, \quad (\text{A4})$$

where r is the ratio of energy density of dark energy to dark matter, and this is just Eq. (15).

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